# 제2장 전력 계산

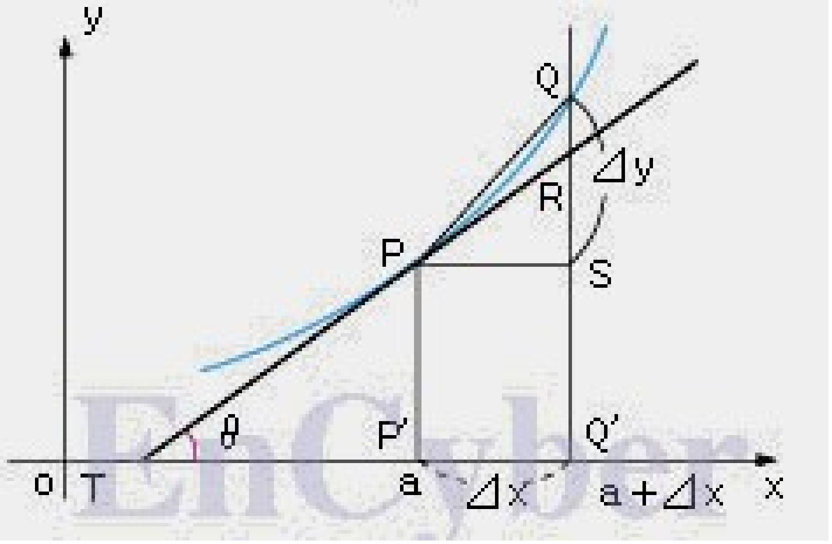
# *Power Computation*

# 2.1 서론

- 전기의 기본 개념 정리
- 전력계산
  - ✓정현파(Sin Wave)
  - ✓비정현파
- OrCAD를 이용하여 전력계산

# 예비학습: 미분(differentiation)

- (x)가 미분 가능인 경우에 y=f(x)라 놓고 x와 y의 증분을 각각 Δx,Δy로 놓고 변화율을 구함



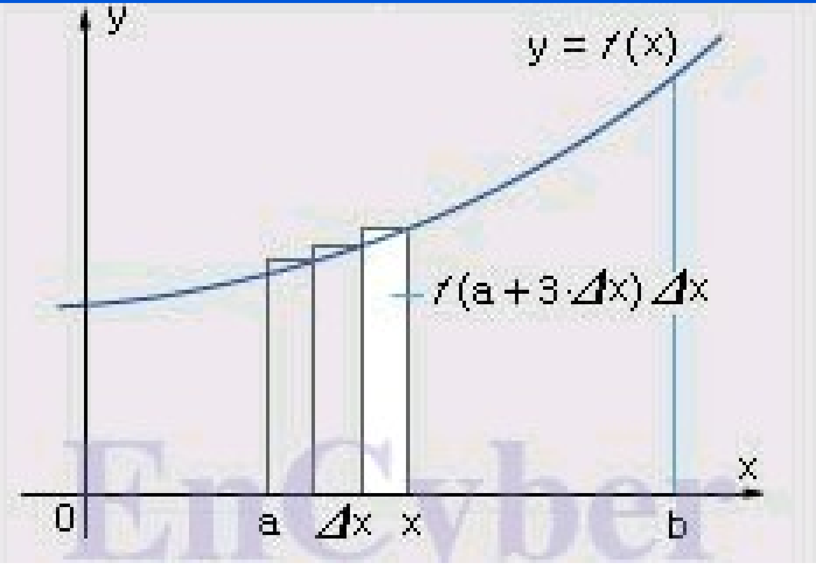

$$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{[f(x+\Delta x) - f(x)]}{\Delta x}$$

$$= \frac{dy}{dx}$$

# 예비학습: 적분(integral)

- 곡선 f(x)가 있을 때, 이 곡선과x축 및 두 직선 $x_1 = a$, $x_2 = b (b > a)$로 둘러싸인 평면의 넓이를 구하면

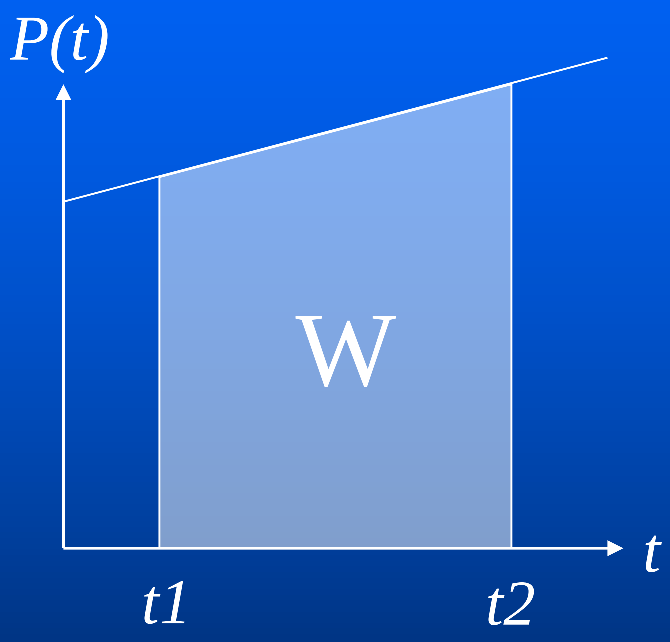



$$\lim_{n\to 0}\left[\sum_{k=1}^{n} f\left(a+k\Delta x\right)\right]\Delta x$$

$$F'(x) = f(x)$$

$$\int_a^b f(x)dx = F(X)$$

# 2.2 전력과 에너지




- 순시전력 (Instantaneous power)

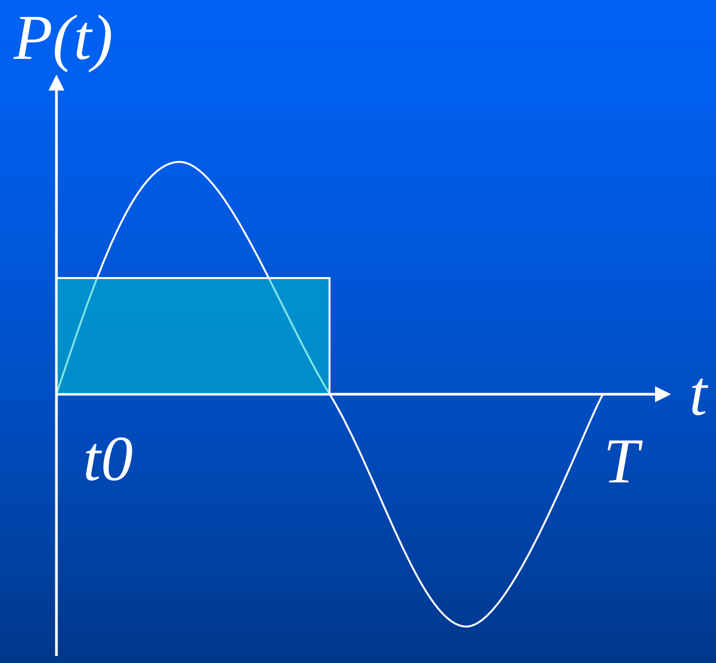
$$p(t) = i(t)v(t)\ [W]$$

- 에너지(Energy)

$$w(t) = \int_{t1}^{t2} p(t)\,dt\ [J]$$

→ *joule[J]= watt [W]× time[s] [KWh]*

# 2.2 전력과 에너지: 평균값(Mean Value)




*정현파는 반주기로 계산
한주기 계산시 0가 됨

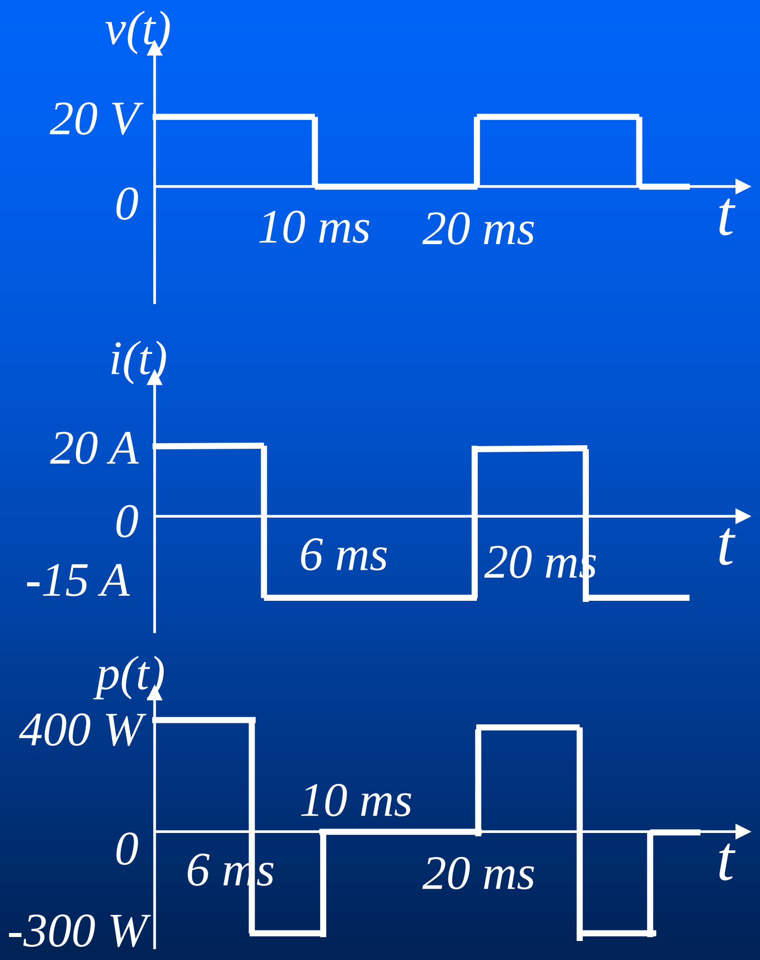
- 전력의 평균값

$$P = \frac{1}{T}\int_{t0}^{t0+T} p(t)\,dt$$

$$= \frac{1}{T}\int_{t0}^{t0+T} v(t)i(t)\,dt$$

$T$ = 주기(period)

# 예제 2-1 전력과 에너지




- 에너지

$$W = \int_{t0}^{t0+T} p(t)\,dt$$

$$= \int_{0}^{0.006} 400\,dt + \int_{0.006}^{0.01} -300\,dt + \int_{0.01}^{0.02} 0\,dt$$

$$= \left[400t\right]_{0}^{0.006} + \left[-300t\right]_{0.006}^{0.01}$$

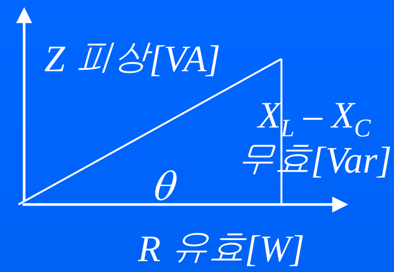
$$= (2.4-0)+(-3+1.8) = 1.2[J]$$

- 평균전력

$$P = \frac{1}{T}\int_{t0}^{t0+T} p(t)\,dt$$

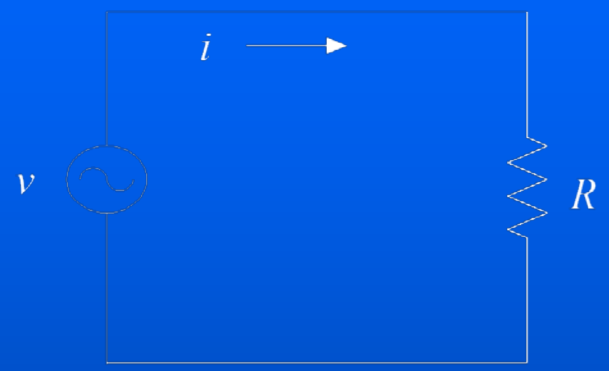
$$= \frac{1}{0.02}[1.2] = 60[W]$$

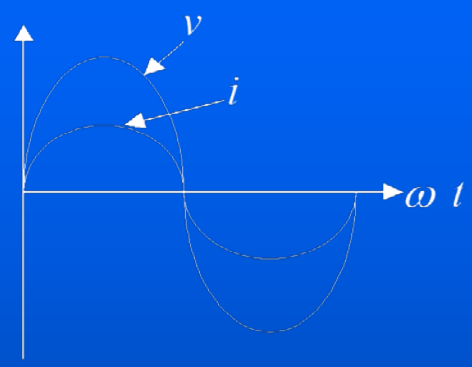
# 중요: 수동소자




| 직류 (주파수 f =0) | | | 교류 (주파수 f >0) | | |
|---|---|---|---|---|---|
| | 기본공식 | 에너지저장 | | 리액턴스 | 임피던스/역율 |
| R 저항 [Ω] | $I = \frac{V}{R}$ | 0 | R [Ω] | $R$ | $Z = \sqrt{R^2 + (X_L - X_C)^2}$ |
| L 인덕턴스 [H] | $V = L\frac{di}{dt}$ | $W = \frac{1}{2}Li^2$ | $X_L$ [Ω] | $X_L = 2\pi f L$ 유도성 | $\cos\theta = \frac{R}{Z} = \frac{\text{유효}}{\text{피상}}$ |
| C 커패시턴스 [F] | $i = C\frac{dv}{dt}$ | $W = \frac{1}{2}Cv^2$ | $X_C$ [Ω] | $X_C = \frac{1}{2\pi f C}$ 용량성 | |

# 예비학습: 저항(R) 회로



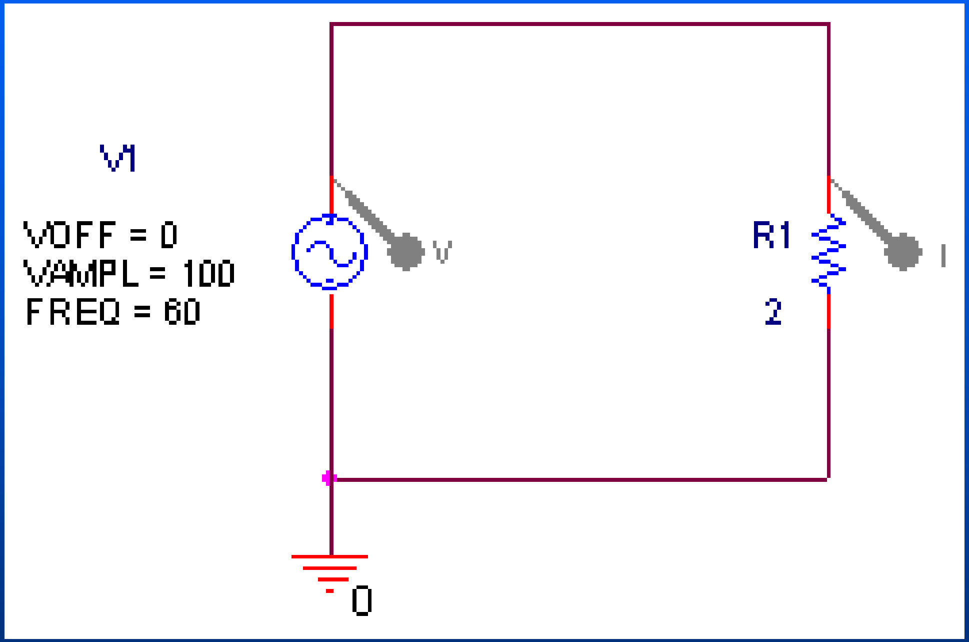




- 전류

$$i = \frac{v}{R}$$

- 전압과 전류의 위상: 동상(same phase)

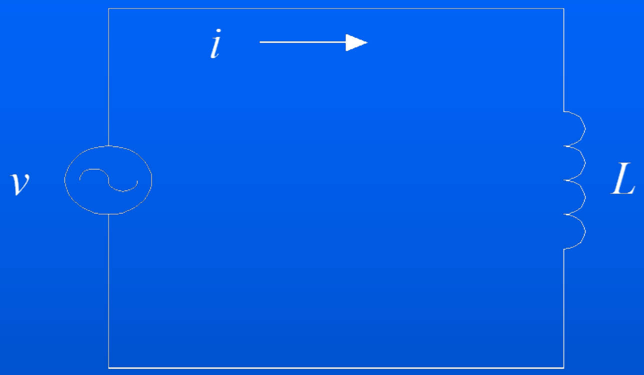
# R 회로: OrCAD Simulation

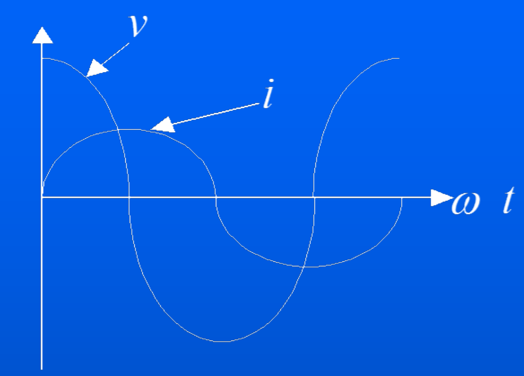



- *Parameter*
  - ✓ *R=2 Ω*
  - ✓ *V1=100 V*
  - ✓ *f=60 Hz*
  - ✓ *Transient Step:*

    *0 0.1 ms 50 ms*

# 예비학습: 인덕터(L) 회로



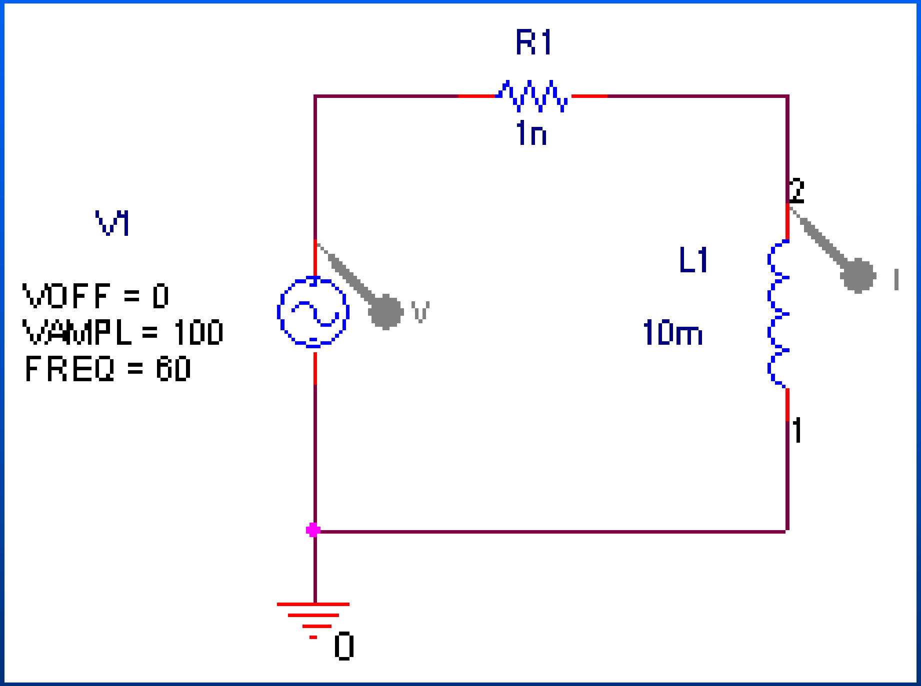

- 전압

$$v = L\frac{di}{dt} \quad = L\frac{d}{dt}I_m \sin\omega t \quad = LI_m \frac{d\sin\omega t}{d\omega t}\frac{d\omega t}{dt}$$

$$= \omega L I_m \cos\omega t \quad = \omega L I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

- 전압과 전류의 위상: 전류가 90°이 뒤짐(lag)

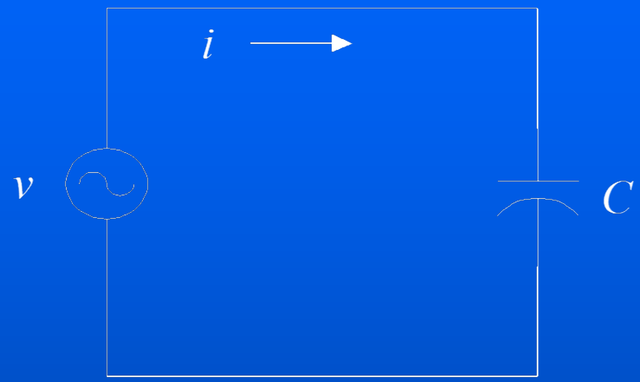
# L 회로: OrCAD Simulation

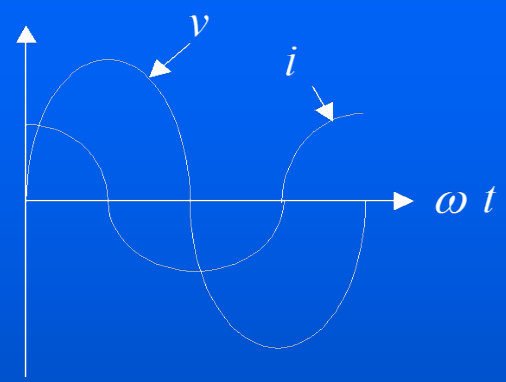



- *Parameter*
  - ✓ *R=1 nΩ*
  - ✓ *L=10mH*
  - ✓ *V1=100 V*
  - ✓ *f=60 Hz*
  - ✓ *Transient Step:*

    *0  0.1 ms  50 ms*

# 예비학습: 커패시터(C) 회로




- 전류

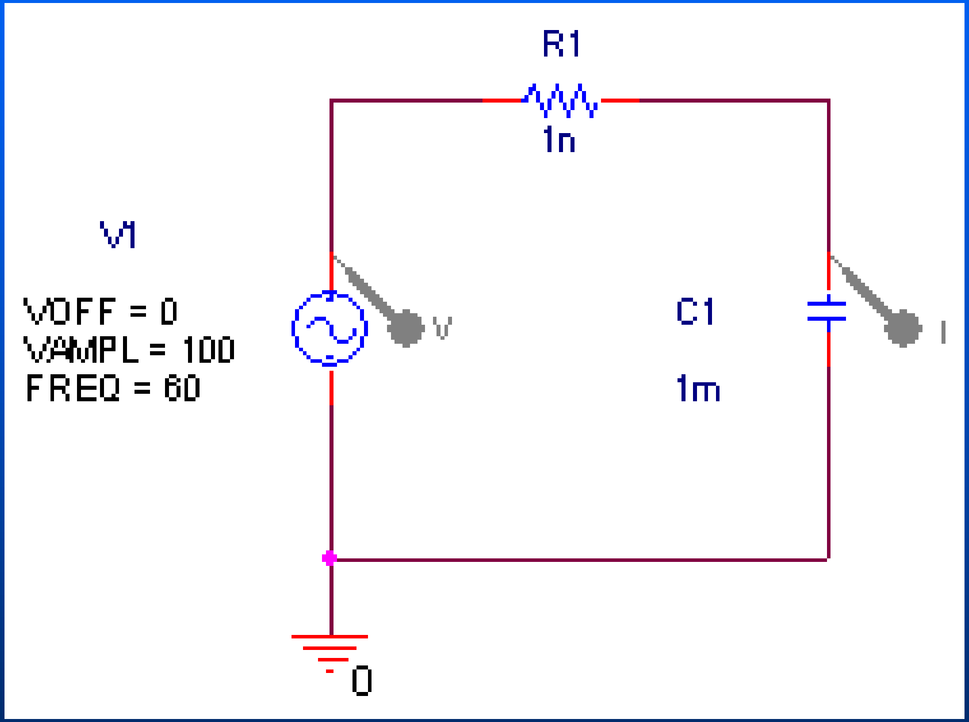
$$\boxed{\because Q = Cv}$$

$$i = \frac{dQ}{dt} = C\frac{dv}{dt} = C\frac{d}{dt}V_m \sin\omega t = CV_m \frac{d\sin\omega t}{d\omega t}\frac{d\omega t}{dt}$$

$$= \omega C V_m \cos\omega t = \omega C V_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

- 전압과 전류의 위상: 전류가 90° 앞섬(lead)

# C 회로: OrCAD Simulation




- *Parameter*
  - ✓ *R=2 Ω*
  - ✓ *C=1mF*
  - ✓ *V1=100 V*
  - ✓ *f=60 Hz*
  - ✓ *Transient Step:*

    *0  0.1 ms 50 ms*

# 2.3 인덕터와 커패시터: Inductor

- 인턱터의 저장에너지 (Storage Energy)

$$w(t) = \int i(t)v(t)\,dt = \int i(t)L\frac{di(t)}{dt}\,dt$$

$$= L\int i(t)di(t) = \frac{1}{2}Li^2(t)\ [J]$$

- 평균소비전력: 전류가 주기함수 일 때
  - ✓ 주기전류: 시작=끝

$$P_L = 0[W],\quad i(t_0+T) = \frac{1}{L}\int_{t_0}^{t_0+T} v_L(t)\,dt + i_0$$

$$i(t_0+T) - i_0 = \frac{1}{L}\int_{t_0}^{t_0+T} v_L(t)\,dt = 0$$

$$\boxed{\begin{aligned} &\because v_L(t) = L\frac{di}{dt} \\ &i = \frac{1}{L}\int_{t_0}^{t_0+T} v_L(t)\,dt + i_0 \end{aligned}}$$

# 2.3 인덕터와 커패시터: Capacitor

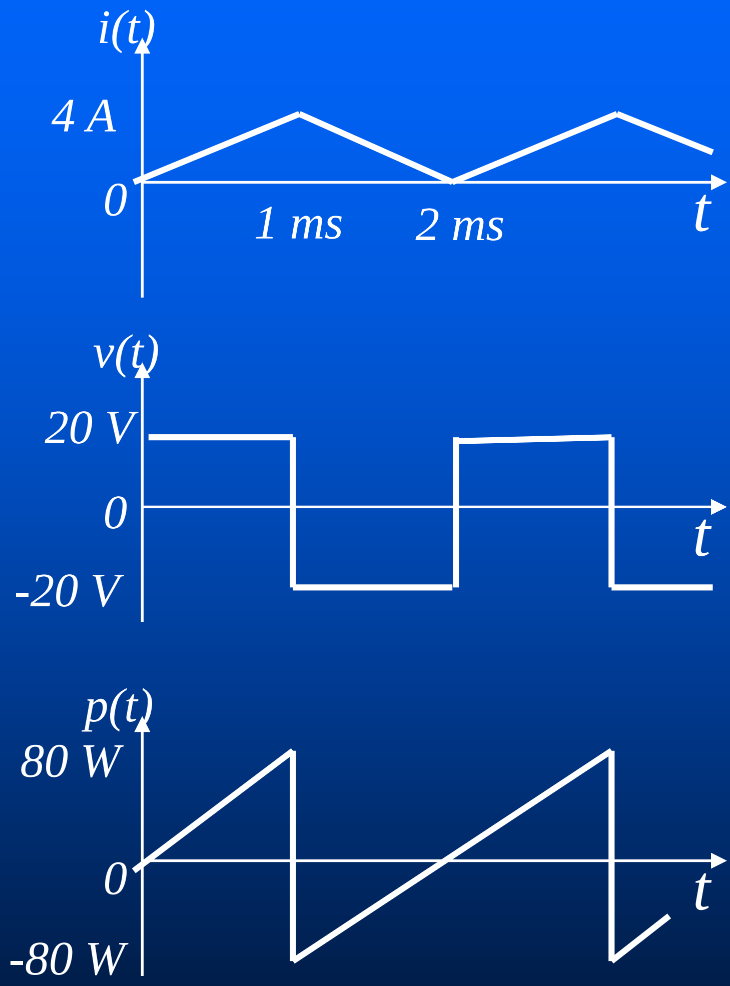
- 캐패시터의 저장에너지 (Storage Energy)

$$w(t) = \int v(t)i(t)\,dt = \int v(t)C\frac{dv(t)}{dt}dt$$
$$= C\int v(t)dv(t) = \frac{1}{2}Cv^2(t)\ [J]$$

- 평균소비전력: 전압이 주기함수 일때
  - ✓ 주기전압: 시작=끝

$$P_C = 0[W],\ v(t_0+T) = \frac{1}{C}\int_{t_0}^{t_0+T} i_C(t)\,dt + v_0$$
$$v(t_0+T) - v_0 = \frac{1}{C}\int_{t_0}^{t_0+T} i_C(t)\,dt = 0$$

# 예제 2-2 인덕터의 전력과 전압





- 전압

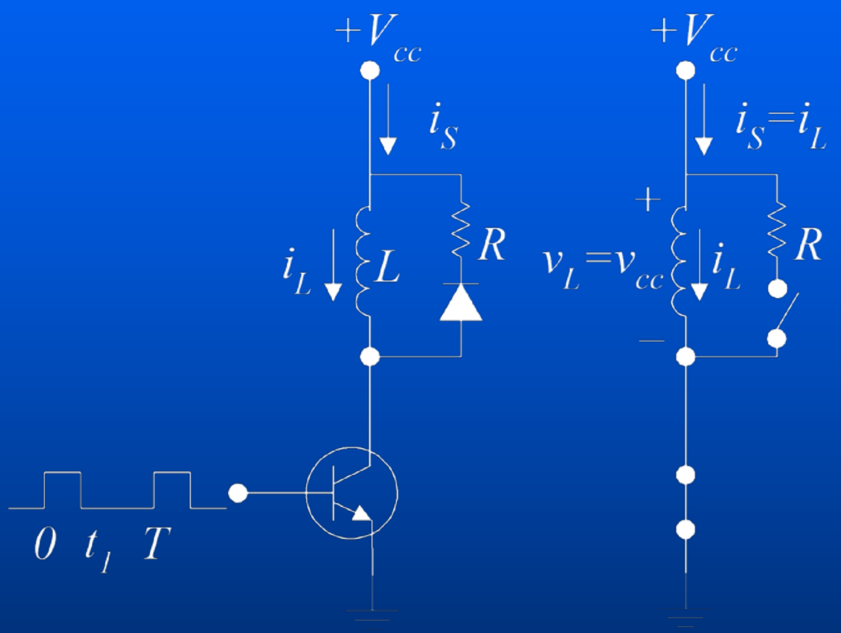
$$v(t) = L\frac{di}{dt} = 0.005\frac{4-0}{0.001-0} = 20[V]$$

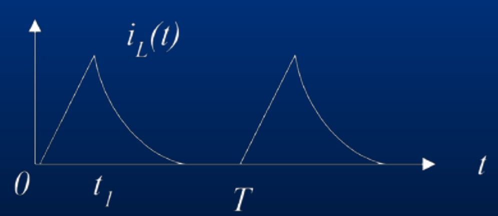
- 전력

$$p(t) = i(t)v(t) = 4\times 20 = 80[W]$$

$$at\ 1mS$$

# 2.4 에너지 회생: Transistor On




$$0 < t < t_1$$

$$v_L = V_{CC} = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{V_{CC}}{L}$$

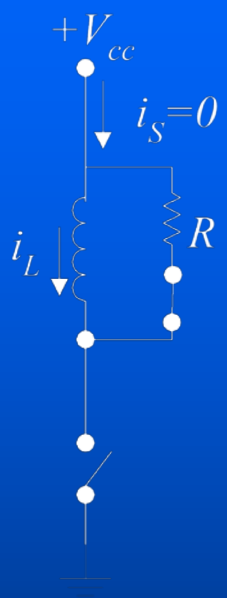
$$i_L(t) = \frac{V_{CC}}{L}\int dt + i_L(0+)$$

$$= \frac{V_{CC}}{L}\int 1dt + 0$$

$$= \frac{V_{CC}}{L}t$$

$$i_S(t) = i_L(t)$$

# 2.4 에너지 회생(2): Transistor Off



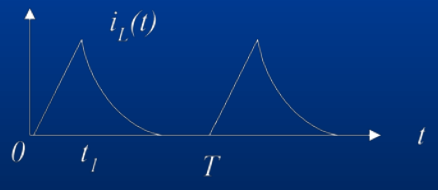

$t_1 < t < T$

$$L\frac{di_L}{d\lambda} + Ri_L = 0 \quad \boxed{by\ \ Laplace\,Transform}$$

$$L\big(SI(S) - i_0(0+)\big) + RI(S) = 0 \quad \boxed{where\ \ i_0(0+) = \frac{V_{CC}t_1}{L}}$$

$$SI(S) + \frac{R}{L}I(S) = \frac{V_{CC}t_1}{L}, \quad I(S)\left(S + \frac{R}{L}\right) = \frac{V_{CC}t_1}{L}$$

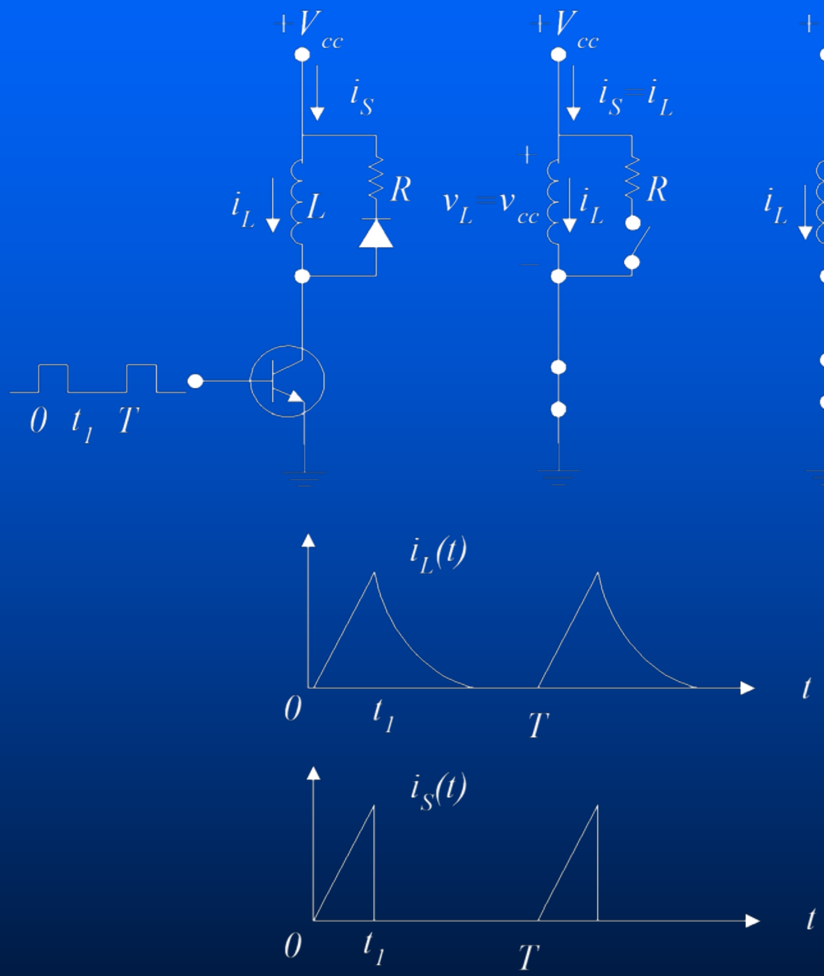
$$I(S) = \frac{V_{CC}t_1}{L}\frac{1}{\left(S + \frac{R}{L}\right)} \quad \boxed{by\ \ Inverse\,Laplace\,Transform}$$




$$i_L(t) = \frac{V_{CC}t_1}{L}e^{-\frac{R}{L}\lambda}, \ \lambda = t - t_1, \quad i_L(t) = \frac{V_{CC}t_1}{L}e^{-\frac{R}{L}t - t_1}$$

$$i_L(t) = \frac{V_{CC}t_1}{L}e^{-\frac{(t-t_1)}{\tau}} \quad \boxed{where\ \ \tau = \frac{L}{R}}$$

# 2.4 에너지 회생(3): Summary



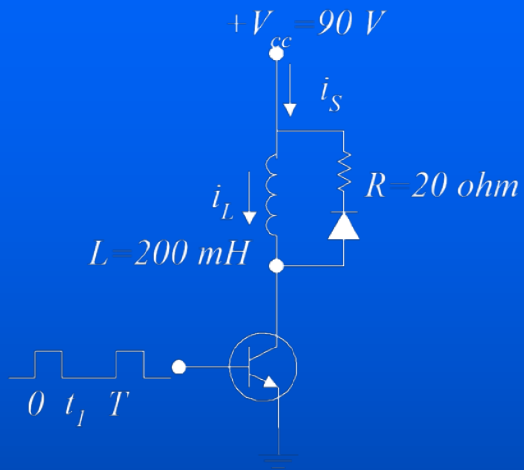

$$0 < t < t_1$$

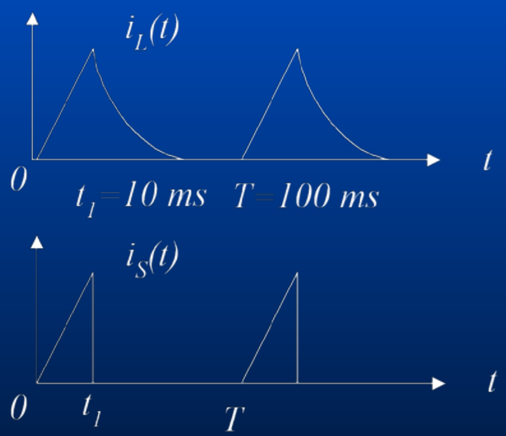
$$i_L(t) = \frac{V_{CC}t}{L}$$

$$t_1 < t < T$$

$$i_L(t) = \frac{V_{CC}t_1}{L} e^{-\frac{(t-t_1)}{\tau}}$$

# 예제 2-3 회생 에너지




- 인덕터 전류

$$i_L(t) = \frac{V_{CC}t}{L} = \frac{90t}{0.2} = 450t \quad at \ \ 0 < t < 10mS$$

- 인덕터 첨두전류

$$i_L(t_1) = \frac{V_{CC}t_1}{L} = 450(0.01) = 4.5\,A$$

- 인덕터 에너지

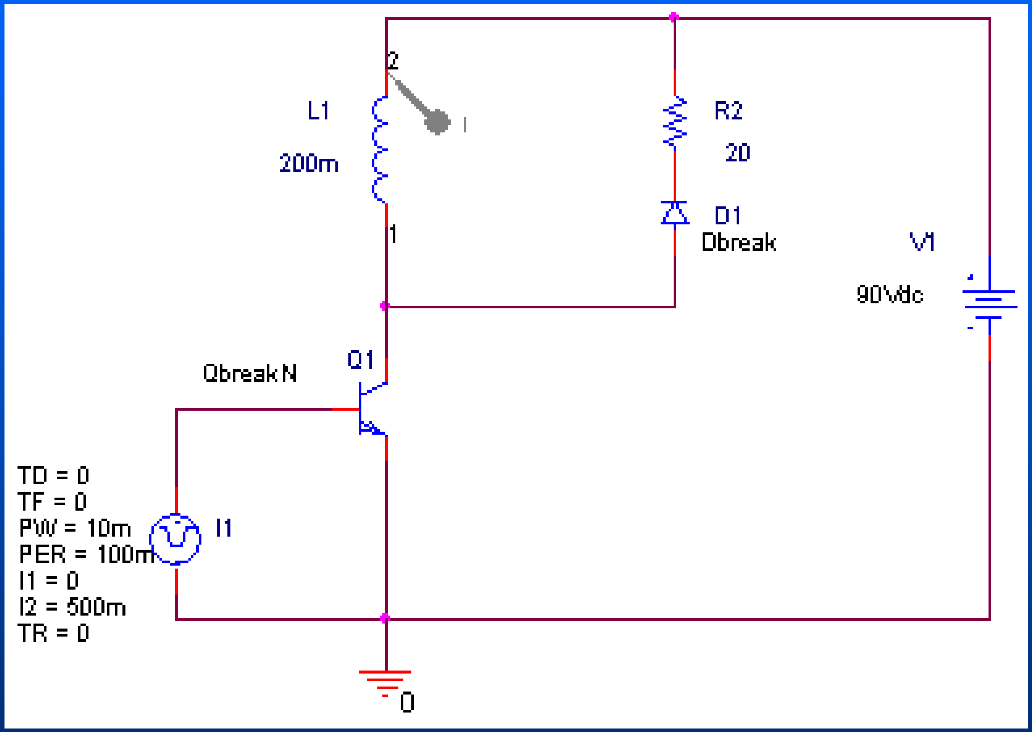
$$W_L = \frac{1}{2}Li^2(t_1) = \frac{1}{2}(0.2)(4.5)^2 = 2.025\,J$$

- 평균 소비전력

$$W_R = W_L = 2.025\,J$$

$$P_R = \frac{W_R}{T} = \frac{2.025\,J}{0.1S} = 20.25\,W$$

# 예제2-3: OrCAD Simulation

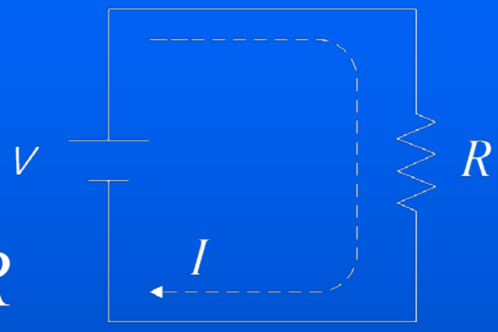


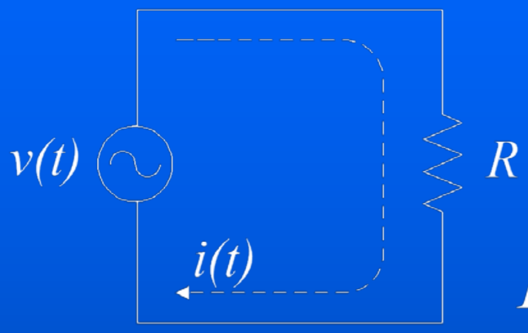

- *Parameter*
  - ✓ *R=20 Ω*
  - ✓ *L=200 mH*
  - ✓ *Ipulse*
  - ✓ *I1=1000mA*
  - ✓ *V1=90 V*
  - ✓ *Transient Step:*

    *0 0.1 ms 300 ms*

# 2.5 실효값(rms: root mean square)

$$P = I^2 R$$



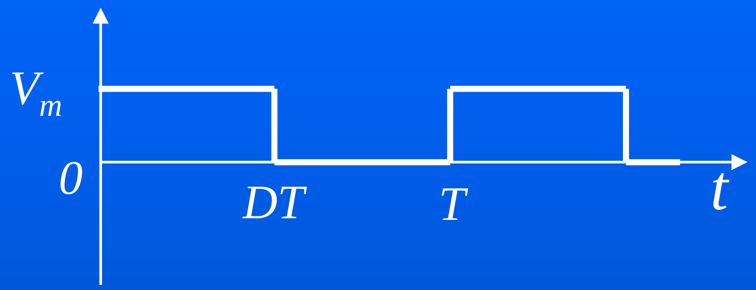

$$p(t) = i^2 R$$

- 직류전력의 평균값=교류전력의 평균값

$$P = I^2 R = \frac{1}{T}\int_0^T i^2 R(t)\, dt$$

$$I = \sqrt{\frac{1}{T}\int_0^T i^2(t)\, dt}$$

# 예제 2-4 펄스파의 평균값



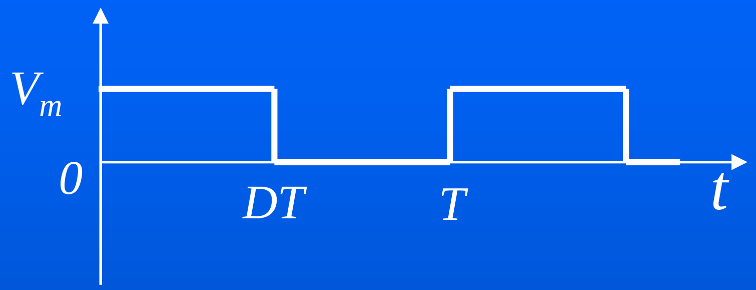

- 듀티비(duty ratio) =On시간/주기(T)

$$v(t) = \begin{cases} V_m & 0 < t < DT \\ 0 & DT < t < T \end{cases}$$

$$V_0 = \frac{1}{T}\int_0^T v(t)\,dt \quad = \frac{1}{T}\left(\int_0^{DT} V_m\,dt + \int_{DT}^{T} 0\,dt\right)$$

$$= \frac{V_m}{T}\left[t\right]_0^{DT} \quad = \frac{V_m}{T}\left[DT - 0\right] \quad = V_m D$$

# 예제 2-4 펄스파의 실효값



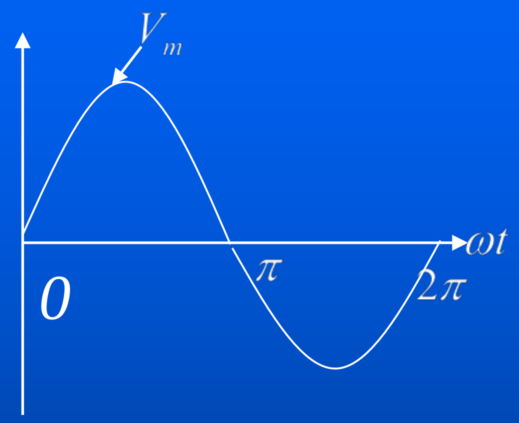

- 듀티비(duty ratio) =On시간/주기(T)

$$v(t) = \begin{cases} V_m & 0 < t < DT \\ 0 & DT < t < T \end{cases}$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt} = \sqrt{\frac{1}{T}\left(\int_0^{DT} V_m^{\ 2}\,dt + \int_{DT}^{T} 0\,dt\right)}$$

$$= \sqrt{\frac{V_m^{\ 2}}{T}\left[t\right]_0^{DT}} = \sqrt{\frac{V_m^{\ 2}}{T}\left[DT - 0\right]} = V_m\sqrt{D}$$

# 예제 2-5 정현파의 평균값




$$V_0 = \frac{1}{\pi}\int_0^{\pi}(V_m \sin\omega t)\,d(\omega t)$$

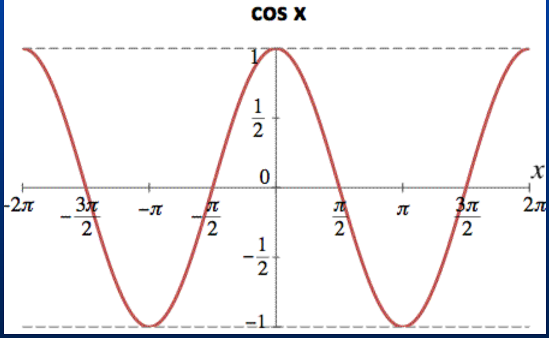
$$= \frac{V_m}{\pi}\left[-\cos\omega t\right]_0^{\pi}$$

$$= \frac{V_m}{\pi}\left\{-\cos\pi - (-\cos 0)\right\}$$

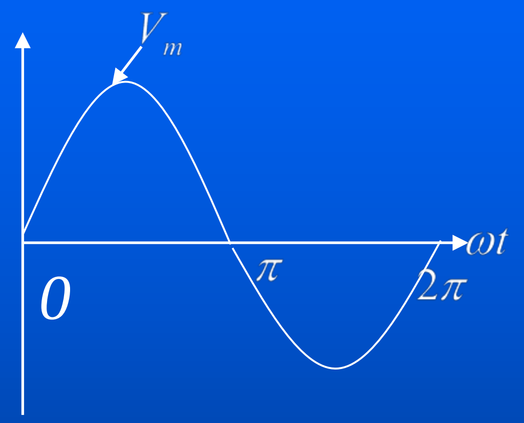
$$= \frac{V_m}{\pi}\left\{1 - (-1)\right\}$$

$$= \frac{2V_m}{\pi}$$

❖ 사인파는 반주기로 계산
한주기 계산시 0가 됨



# 예제 2-5 정현파의 실효값




❖ 사인파는 반주기로 계산
한주기 계산시 0가 됨

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi}\left(V_m \sin\omega t\right)^2 d(\omega t)}$$

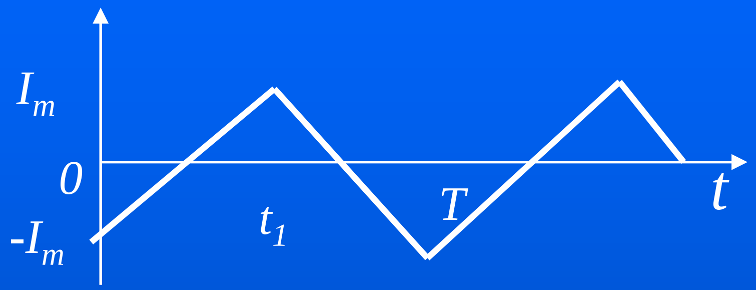
$$\boxed{where\left(\sin^2 x = \frac{1-\cos 2x}{2}\right)}$$

$$= \sqrt{\frac{V_m^2}{\pi}\int_0^{\pi}\frac{1-\cos 2\omega t}{2} d(\omega t)}$$

$$= \sqrt{\frac{V_m^2}{\pi}\left[\frac{\omega t}{2} - \frac{\sin 2\omega t}{4}\right]_0^{\pi}}$$

$$= \sqrt{\frac{V_m^2}{\pi}\left(\frac{\pi}{2} - \frac{\sin 2\pi}{4} - \frac{0}{2} + \frac{\sin 0}{4}\right)} = \frac{V_m}{\sqrt{2}}$$

# 예제 2-8 삼각파의 실효값



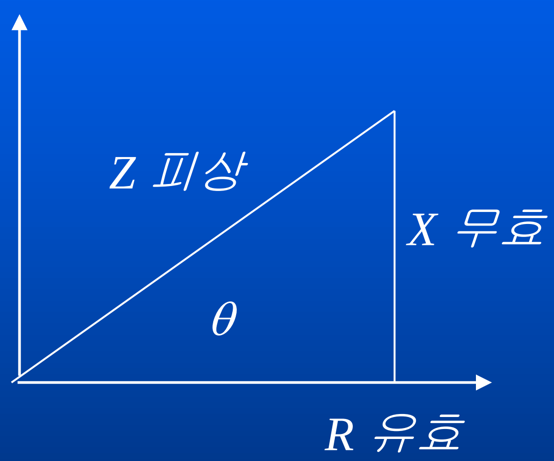

- 듀티비(duty ratio)
  =온시간/주기(T)

$$i(t)=\begin{cases}\dfrac{2I_m}{t_1}t-I_m & 0<t<t_1\\[2ex] \dfrac{-2I_m}{T-t_1}t+\dfrac{I_m(T+t_1)}{T-t_1} & t_1<t<T\end{cases}$$

$$I_{rms}=\sqrt{\frac{1}{T}\int_0^{t_1}\left(\frac{2I_m}{t_1}t-I_m\right)^2dt+\sqrt{\frac{1}{T}\int_{t_1}^{T}\left(\frac{-2I_m}{T-t_1}t+\frac{I_m(T+t_1)}{T-t_1}\right)^2dt}}$$

$$=\frac{I_m}{\sqrt{3}}$$

# 2.6 피상전력과 역률




- 피상전력(Apparent power)

$$S = V_{rms} I_{rms} = \sqrt{P^2 + Q^2}$$

- 역률(Power factor)

$$pf = \cos\theta = \frac{P}{S} = \frac{P}{V_{rms} I_{rms}}$$

# 2.7 정현교류회로의 전력계산(1)

$$v(t) = V_m \cos(\omega t + \theta),\ \ i(t) = I_m \cos(\omega t + \phi)$$

$$p(t) = v(t)i(t) = [V_m \cos(\omega t + \theta)][I_m \cos(\omega t + \phi)]$$

$$\left( \begin{aligned} where\ \ \cos A \cos B &= \frac{1}{2}[\cos(A+B) + \cos(A-B)] \\ \sin A \sin B &= \frac{1}{2}[-\cos(A+B) + \cos(A-B)] \\ \sin A \cos B &= \frac{1}{2}[\sin(A+B) + \sin(A-B)] \end{aligned} \right)$$

$$p(t) = \frac{V_m I_m}{2}[\cos(\omega t + \theta + \omega t + \phi) + \cos(\omega t + \theta - \omega t - \phi)]$$

$$= \frac{V_m I_m}{2}[\cos(2\omega t + \theta + \phi) + \cos(\theta - \phi)]$$

# 2.7 정현교류회로의 평균전력계산(2)

$$p(t) = \frac{V_m I_m}{2}\left[\cos(2\omega t + \theta + \phi) + \cos(\theta - \phi)\right]$$

*한주기 적분은 0

$$P = \frac{1}{T}\int_0^T p(t)dt = \frac{V_m I_m}{2T}\int_0^T \left[\cos(2\omega t + \theta + \phi) + \cos(\theta - \phi)\right]dt$$

- 임의소자의 피상전력

$$p(t) = \frac{V_m I_m}{2T}\int_0^T \left[\cos(\theta - \phi)\right]dt = \frac{V_m I_m}{2T}\cos(\theta - \phi)\left[t\right]_0^T$$

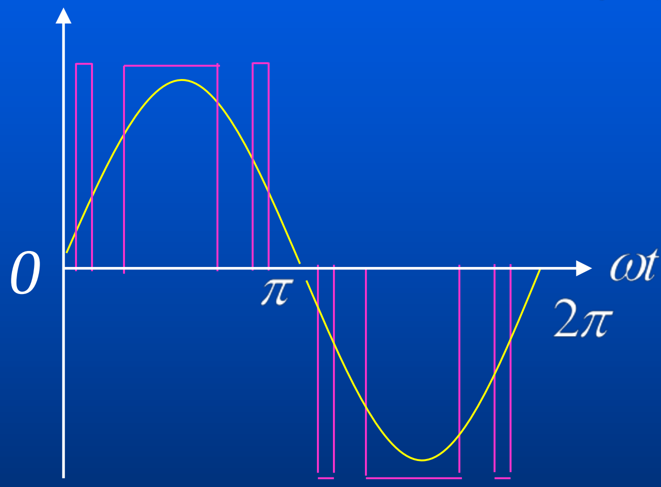
$$= \frac{V_m I_m T}{2T}\cos(\theta - \phi) = \frac{\sqrt{2}V_{rms}\sqrt{2}I_{rms}}{2}\cos(\theta - \phi)$$

$$= V_{rms} I_{rms}\cos(\theta - \phi)$$

# 2.8 비정현파 주기함수의 전력계산

- Fourier Series(푸리에 급수)

$$f(t) = a_0 + \sum_{n=1}^{\infty}\left[a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)\right]$$




$$a_0 = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)dt$$

$$a_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\cos(n\omega_0 t)dt$$

$$b_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\sin(n\omega_0 t)dt$$

# 2.8 비정현파 주기함수의 전력계산(2)

- 정현함수로 합치면

$$f(t) = a_0 + \sum_{n=1}^{\infty}\left[a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)\right]$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$$

$$C_n = \sqrt{a_n^2 + b_n^2}$$

$$\theta_n = \tan^{-1}\frac{-b_n}{a_n}$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} C_n \sin(n\omega_0 t + \theta_n)$$

$$C_n = \sqrt{a_n^2 + b_n^2}$$

$$\theta_n = \tan^{-1}\frac{a_n}{b_n}$$

# 2.8 비정현파 주기함수: 평균전력

$$v(t) = V_0 + \sum_{n=1}^{\infty} V_n \cos\left(n\omega_0 t + \theta_n\right)$$

$$i(t) = I_0 + \sum_{n=1}^{\infty} I_n \sin\left(n\omega_0 t + \theta_n\right)$$

$$P = \frac{1}{T}\int_0^T v(t)i(t)dt$$

$$P = \sum_{n=1}^{\infty} P_n = V_0 I_0 + \sum_{n=1}^{\infty} V_{n,rms} I_{n,rms} \sin\left(n\omega_0 t + \theta_n\right)$$

$$P = \sum_{n=1}^{\infty} P_n = V_0 I_0 + \sum_{n=1}^{\infty} \frac{V_{n,\max} I_{n,\max}}{2} \sin\left(n\omega_0 t + \theta_n\right)$$

$$\because V_{n,\max} I_{n,\max} = \left(\sqrt{2}\right)^2 V_{n,rms} I_{n,rms}$$

# 2.8 비정현파 주기함수: 왜곡율

$$P=\sum_{n=1}^{\infty}P_n=V_0I_0+\sum_{n=1}^{\infty}\frac{V_{n,\max}I_{n,\max}}{2}\sin(n\omega_0 t+\theta_n)$$

$$P=V_0I_0+\frac{V_{1,\max}I_{1,\max}}{2}\sin(\omega_0 t+\theta_1)+\frac{V_{2,\max}I_{2,\max}}{2}\sin(2\omega_0 t+\theta_2)+\ldots$$

- 왜곡율(Distortion factor)

$$DF=\frac{I_{1,rms}}{I_{rms}}$$

- 전고조파왜곡(Total Harmonic Distortion)

$$THD=\sqrt{\frac{\sum_{n\neq 1}I_{n,rms}^2}{I_{1,rms}^2}}=\frac{\sqrt{\sum_{n\neq 1}I_{n,rms}^2}}{I_{1,rms}}=\sqrt{\frac{I_{rms}^2-I_{1,rms}^2}{I_{1,rms}^2}}$$

# 예제 2.10 정현전원과 비선형 부하

$$v(t) = 100\cos(377t)$$

$$i(t) = 8 + 15\cos(377t + 30) + 6\cos(2.377t + 45) + 2\cos(3.377t + 60)$$

(a) 부하의 소비 전력

$$P = \sum_{n=1}^{\infty} P_n = V_0 I_0 + \sum_{n=1}^{\infty} V_{n,rms} I_{n,rms} \sin\left(n\omega_0 t + \theta_n\right)$$

$$= (0)(8) + \left(\frac{100}{\sqrt{2}}\right)\left(\frac{15}{\sqrt{2}}\right)\cos(30^\circ) + (0)\left(\frac{6}{\sqrt{2}}\right)\cos(45^\circ) + (0)\left(\frac{2}{\sqrt{2}}\right)\cos(60^\circ)$$

$$= \left(\frac{1500}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = 650[W]$$

# 예제 2.10 정현전원과 비선형 부하(2)

$$v(t) = 100\cos(377t)$$

$$i(t) = 8 + 15\cos(377t + 30) + 6\cos(2.377t + 45) + 2\cos(3.377t + 60)$$

(b) 부하의 역율

$$V_{rms} = \frac{100}{\sqrt{2}} = 70.7[V]$$

$$I_{rms} = \sqrt{8^2 + \left(\frac{15}{\sqrt{2}}\right)^2 + \left(\frac{6}{\sqrt{2}}\right)^2 + \left(\frac{2}{\sqrt{2}}\right)^2} = 14[A]$$

$$pf = \frac{p}{V_{rms} I_{rms}} = \frac{650}{(70.7)(14.0)} = 0.66$$

# 예제 2.10 정현전원과 비선형 부하(3)

$$v(t) = 100\cos(377t)$$

$$i(t) = 8 + 15\cos(377t + 30) + 6\cos(2.377t + 45) + 2\cos(3.377t + 60)$$

(c) 부하전류의 왜곡율

$$DF = \frac{I_{1,rms}}{I_{rms}} = \frac{\left(\frac{15}{\sqrt{2}}\right)}{14} = 0.76$$

(d) 부하전류의 전고조파 왜곡율

$$THD = \sqrt{\frac{I_{rms}^2 - I_{1,rms}^2}{I_{1,rms}^2}} = \sqrt{\frac{(14)^2 - \left(\frac{15}{\sqrt{2}}\right)^2}{\left(\frac{15}{\sqrt{2}}\right)^2}} = 0.86$$