

## 제 3 장 반파 정류기

### *Half-wave Rectifier* *AC → DC*

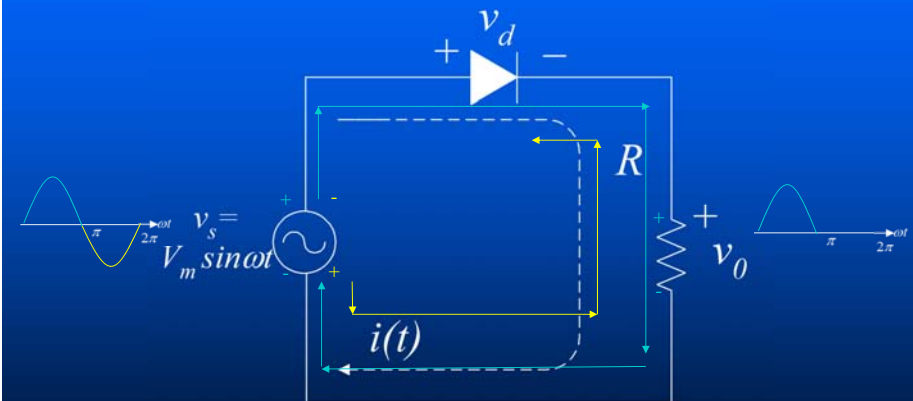
## 3장 반파정류회로의 구성

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- 3.2 저항부하(Resistive Load)
- 3.3 R-L 부하
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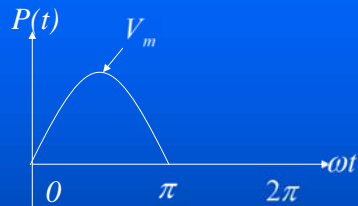
### 3.1 서론

- 정류(Rectification)  
AC → DC로 변환
- 반파 정류
  - 공급되는 전류 평균: 0이 아님
  - 변압기 성능에 문제점 유발 가능
  - 실제로는 전파 전류가 주로 이용
- OrCAD를 이용하여 계산: 가장 간편함

### 3.2 저항부하(Resistive Load)



### 3.2 저항부하: 반파의 평균값



#### ■ 전압의 평균값

$$V_0 = \frac{1}{2\pi} \int_0^\pi V_m \sin \omega t d(\omega t)$$

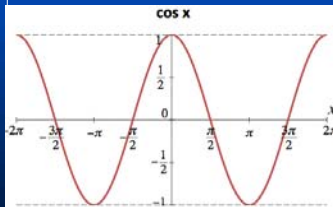
$$= \frac{V_m}{2\pi} [-\cos \omega t]_0^\pi \int \sin dt = -\cos t$$

$$= \frac{V_m}{2\pi} (-\cos \pi - (-\cos 0))$$

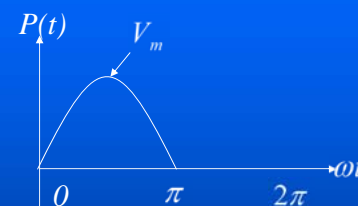
$$= \frac{V_m}{2\pi} (-(-1) + 1) = \frac{V_m}{\pi}$$

#### ■ 전류의 직류 성분

$$I = \frac{V_0}{R} = \frac{V_m}{\pi R}$$



### 3.2 저항부하: 반파의 실효값



$$V_{rms} = \sqrt{\frac{1}{2\pi} \int_0^\pi (V_m \sin \omega t)^2 d(\omega t)}$$

$$\left( \sin^2 \omega t = \frac{1 - \cos 2\omega t}{2} \right)$$

$$= \sqrt{\frac{V_m^2}{2\pi} \left[ \frac{\omega t}{2} - \frac{\sin 2\omega t}{4} \right]_0^\pi}$$

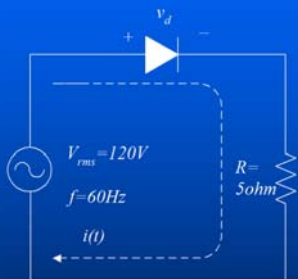
#### ■ 전류의 실효값

$$I_R = \frac{V_{rms}}{R} = \frac{V_m}{2R}$$

$$= \sqrt{\frac{V_m^2}{2\pi} \left( \frac{\pi}{2} - \frac{\sin 2\pi}{4} - \frac{0}{2} + \frac{\sin 0}{4} \right)} = \frac{V_m}{\sqrt{2}}$$

### 예제 3-1 저항부하를 가진 반파정류기(1)

#### (a) 평균 전류



$$V_0 = \frac{1}{2\pi} \int_0^\pi V_m \sin \omega t d(\omega t)$$

$$= \frac{1}{2\pi} \int_0^\pi \sqrt{2}(120) \sin \omega t d(\omega t)$$

$$= \frac{\sqrt{2}(120)}{2\pi} [-\cos \omega t]_0^\pi$$

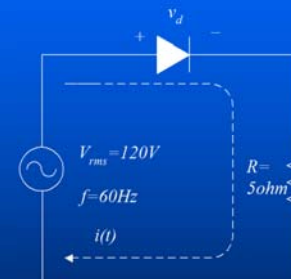
$$= \frac{\sqrt{2}(120)}{2\pi} (-\cos \pi + \cos 0)$$

$$= \frac{\sqrt{2}(120)}{\pi} = 54.02[V]$$

$$\therefore I = \frac{V_0}{R} = \frac{54.02}{5} = 10.8[A]$$

### 예제 3-1 저항부하를 가진 반파정류기(2)

#### (b) 실효 전압



$$V_{rms} = \sqrt{\frac{1}{2\pi} \int_0^\pi (V_m \sin \omega t)^2 d(\omega t)}$$

$$\left( \sin^2 \omega t = \frac{1 - \cos 2\omega t}{2} \right)$$

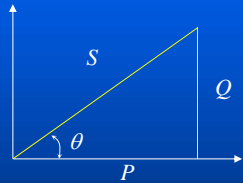
$$= \sqrt{\frac{V_m^2}{2\pi} \left[ \frac{\omega t}{2} - \frac{\sin 2\omega t}{4} \right]_0^\pi}$$

$$= \sqrt{\frac{V_m^2}{2\pi} \left( \frac{\pi}{2} - \frac{\sin 2\pi}{4} - \frac{0}{2} + \frac{\sin 0}{4} \right)}$$

$$= \frac{\sqrt{2}(120)}{2} = 84.9V$$

### 예제 3-1 저항부하를 가진 반파정류기(3)

(c) 역률

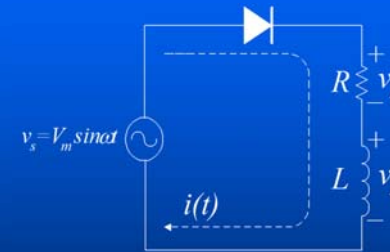


$$P = \frac{V_{rms}^2}{R} = \frac{84.9^2}{4} = 1440 W$$

$$I_{rms} = \frac{V_m}{2R} = \frac{\sqrt{2}(120)}{2(5)} = 17 A$$

$$pf = \frac{p}{V_{rms} I_{rms}} = \frac{1440}{(120)(17)} = 0.707$$

### 3.3 R-L 부하



■ By KVL

$$L \frac{di}{dt} + Ri = V_m \sin \omega t$$

$$\omega L \frac{di}{d\omega t} + Ri = V_m \sin \omega t$$

### 3.3 R-L 부하: 전류계산(1)

$$\omega L \frac{di}{d\omega t} + Ri = V_m \sin \omega t$$

$$\times \frac{1}{\omega L}$$

$$\frac{di}{dt} + \frac{R}{\omega L} i = \frac{V_m}{\omega L} \sin \omega t$$

$$\times e^{\frac{R}{\omega L} \omega t}$$

$$\frac{di}{d\omega t} e^{\frac{R}{\omega L} \omega t} + \frac{R}{\omega L} e^{\frac{R}{\omega L} \omega t} i = \frac{V_m}{\omega L} e^{\frac{R}{\omega L} \omega t} \sin \omega t$$

$$\frac{d}{d\omega t} \left( i e^{\frac{R}{\omega L} \omega t} \right) = \frac{V_m}{\omega L} e^{\frac{R}{\omega L} \omega t} \sin \omega t \quad d(A \cdot B) = dA \cdot B + A \cdot dB$$

$$i e^{\frac{R}{\omega L} \omega t} = \int \frac{V_m}{\omega L} e^{\frac{R}{\omega L} \omega t} \sin \omega t d\omega t + A$$

### 3.3 R-L 부하: 전류계산(2)

$$i e^{\frac{R}{\omega L} \omega t} = \int \frac{V_m}{\omega L} e^{\frac{R}{\omega L} \omega t} \sin \omega t d\omega t + A \quad \times e^{-\frac{R}{\omega L} \omega t}$$

$$i = e^{-\frac{R}{\omega L} \omega t} \frac{V_m}{\omega L} \int e^{\frac{R}{\omega L} \omega t} \sin \omega t d\omega t + A e^{-\frac{R}{\omega L} \omega t}$$

$$\int e^{au} \sin bu du = \frac{e^{au} (a \sin bu - b \cos bu)}{a^2 + b^2}$$

$$i = e^{-\frac{R}{\omega L} \omega t} \frac{V_m}{\omega L} \frac{e^{\left(\frac{R}{\omega L}\right) \omega t} \left( \left( \frac{R}{\omega L} \right) \sin \omega t - \cos \omega t \right)}{\left( \frac{R}{\omega L} \right)^2 + 1^2} + A e^{-\frac{R}{\omega L} \omega t}$$

### 3.3 R-L 부하: 전류계산(3)

$$i = e^{-\frac{R}{\omega L} \omega t} \frac{V_m}{\omega L} \frac{e^{\left(\frac{R}{\omega L}\right) \omega t} \left( \left( \frac{R}{\omega L} \right) \sin \omega t - \cos \omega t \right)}{\left( \frac{R}{\omega L} \right)^2 + 1^2} + A e^{-\frac{R}{\omega L} \omega t}$$

$$= \frac{V_m}{\omega L} \frac{\left( \left( \frac{R}{\omega L} \right) \sin \omega t - \cos \omega t \right)}{\left( \frac{R^2 + (\omega L)^2}{(\omega L)^2} \right)} + A e^{-\frac{R}{\omega L} \omega t}$$

$$= \frac{V_m}{(R^2 + (\omega L)^2)} (R \sin \omega t - \omega L \cos \omega t) + A e^{-\frac{R}{\omega L} \omega t}$$

### 3.3 R-L 부하: 전류계산(4)

$$i = \frac{V_m}{(R^2 + (\omega L)^2)} (R \sin \omega t - \omega L \cos \omega t) + A e^{-\frac{R}{\omega L} \omega t}$$

$$a \cos x + b \sin x = \sqrt{a^2 + b^2} \sin(x + \delta), \delta = \tan^{-1} \frac{a}{b}$$

$$i = \frac{V_m \sqrt{(R^2 + (\omega L)^2)}}{(R^2 + (\omega L)^2)} \sin(\omega t - \theta) + A e^{-\frac{R}{\omega L} \omega t}$$

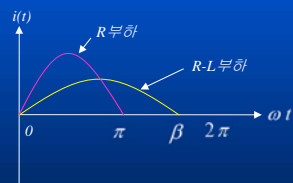
$$= \frac{V_m}{\sqrt{(R^2 + (\omega L)^2)}} \sin(\omega t - \theta) + A e^{-\frac{R}{\omega L} \omega t} = \frac{V_m}{Z} \sin(\omega t - \theta) + A e^{-\frac{R}{\omega L} \omega t}$$

$$\text{where } Z = \sqrt{(R^2 + (\omega L)^2)}, \theta = \tan^{-1} \frac{\omega L}{R}$$

### 3.3 R-L 부하: 전류계산(5)

$$i = \frac{V_m}{Z} \sin(\omega t - \theta) + A e^{-\frac{R}{\omega L} \omega t}$$

- A를 구하기 위한 초기조건  $\omega t=0$  일 때  $i=0$



$$A e^{-0} = 0 - \frac{V_m}{Z} \sin(0 - \theta) = \frac{V_m}{Z} \sin(\theta)$$

$$\therefore i = \frac{V_m}{Z} \left[ \sin(\omega t - \theta) + \sin \theta e^{-\frac{R}{L} t} \right]$$

### 3.3 R-L 부하: 정상과 과도

- By KVL

$$L \frac{di}{dt} + Ri = V_m \sin \omega t$$

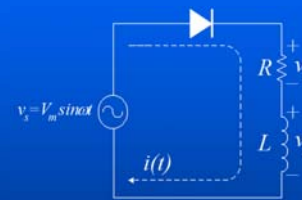
- 전류값

$$i(t) = \frac{V_m}{Z} \left[ \sin(\omega t - \theta) + \sin \theta \cdot e^{-\frac{R}{L} t} \right]$$

- 전류=정상치+과도치

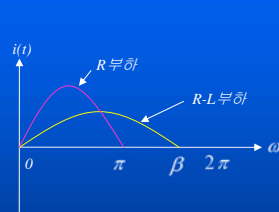
$$i(t) = i_f(t) + i_n(t)$$

$$\text{*여기서 } i_f = \frac{V_m}{Z} \sin(\omega t - \theta), i_n = \frac{V_m}{Z} \sin \theta e^{-\frac{R}{L} t}$$



### 3.3 R-L 부하: 소호각 계산

#### ■ $\beta$ : 소호각(Extinction Angle)



$$i(t) = \frac{V_m}{Z} \left[ \sin(\omega t - \theta) + \sin \theta \cdot e^{-\frac{R}{L}t} \right]$$

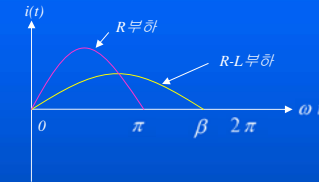
$$i(\beta) = 0 = \frac{V_m}{Z} \left[ \sin(\beta t - \theta) + \sin \theta \cdot e^{-\frac{R\beta}{L\omega}} \right]$$

$$\therefore \sin(\beta t - \theta) + \sin \theta \cdot e^{-\frac{R\beta}{L\omega}} = 0$$

#### ■ $\beta$ 의 계산

- 수치기법(Numerical Analysis) 이용 계산
- OrCAD를 이용하여 계산: 가장 간편함

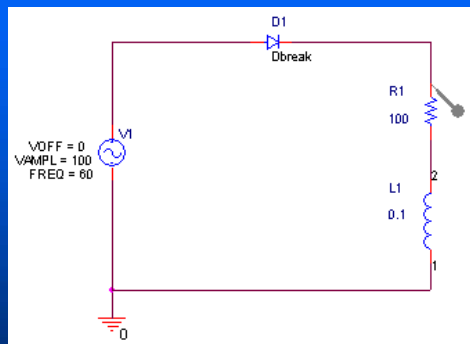
### 3.3 R-L 부하: 종합



$$i(\omega t) = \begin{cases} \frac{V_m}{Z} \left[ \sin(\omega t - \theta) + \sin \theta \cdot e^{-\frac{R}{L}t} \right] & \text{at } 0 \leq \omega t \leq \beta \\ 0 & \text{at } \beta \leq \omega t \leq 2\pi \end{cases}$$

\*여기서  $Z = \sqrt{R^2 + (\omega L)^2}$ ,  $\theta = \tan^{-1}\left(\frac{L}{R}\right)$

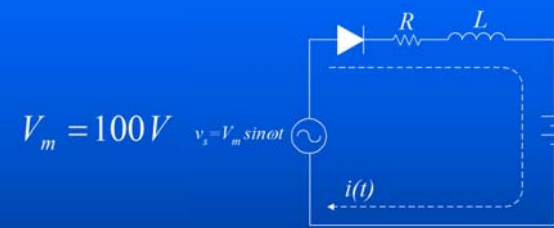
### 예제 3-3 R-L 부하를 가진 반파정류기



#### ■ Parameter

- ✓  $R=100 \Omega$
- ✓  $L=0.1 H$
- ✓  $\omega=377 \text{ rad/s}$ ,  
 $f=\omega/2\pi=60 \text{ Hz}$
- ✓  $V_m=100 V$
- ✓ Transient:  
 $0 \quad 0.01 \text{ ms} \quad 50 \text{ ms}$

### 3.5 R-L 기전력부하



$$V_m = 100 V \quad v_s = V_m \sin \omega t$$

$$i = \frac{V_m - V_{dc}}{Z}$$

$$= \frac{100 - 0}{1} = 100 A$$

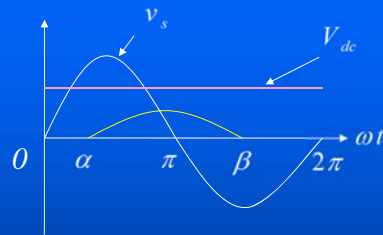
$$i = \frac{100 - 50}{1} = 50 A$$

#### ■ By KVL

$$V_m \sin \omega t = Ri + L \frac{di}{dt} + V_{dc}$$

$$i = \frac{100 - 100}{1} = 0$$

### 3.5 R-L 기전력: 전류



$$V_m \sin \alpha = V_{dc}$$

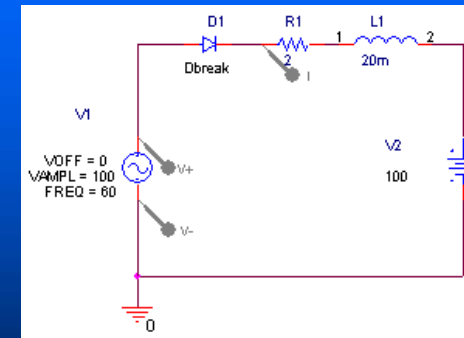
$$\therefore \alpha = \sin^{-1} \left( \frac{V_{dc}}{V_m} \right)$$

#### ■ 전류

$$i(\omega t) = \frac{V_m}{Z} \sin(\omega t - \theta) - \frac{V_{dc}}{R} + \left( -\frac{V_m}{Z} \sin(\alpha - \theta) + \frac{V_{dc}}{R} \right) e^{-\frac{\alpha}{\omega \tau}}$$

at  $\alpha \leq \omega t \leq \beta$

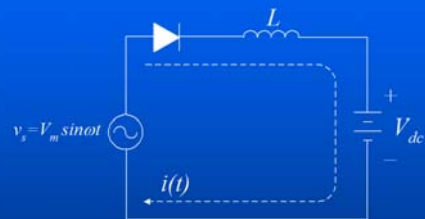
### 예제 3-5 R-L 기전력부하 반파정류기



#### ■ Parameter

- ✓  $R=2 \Omega$
- ✓  $L=20 \text{ mH}$
- ✓  $f=\omega/2\pi=60 \text{ Hz}$
- ✓  $V_m=169.7 \text{ V}$
- ✓  $V_{dc}=100 \text{ V}$
- ✓ Transient:  
0 0.01m 50ms

### 3.6 인덕터 기전력부하



#### ■ By KVL

$$V_m \sin \omega t = L \frac{di}{dt} + V_{dc}$$

$$V_m \sin \omega t = \omega L \frac{di}{d\omega t} + V_{dc}$$

### 3.6 인덕터 기전력부하: 전류

$$V_m \sin \omega t = \omega L \frac{di}{d\omega t} + V_{dc}$$

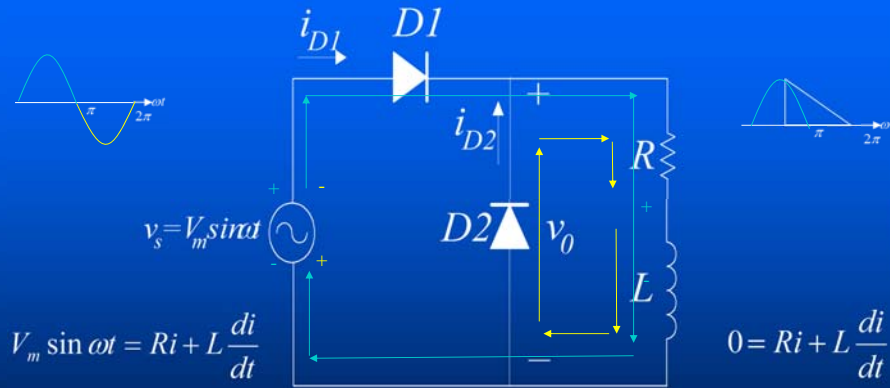
$$\frac{di}{d\omega t} = \frac{V_m \sin \omega t - V_{dc}}{\omega L}$$

$$i(\omega t) = \frac{1}{\omega L} \int_0^\omega V_m \sin \omega t d\omega t - \frac{1}{\omega L} \int_0^\omega V_{dc} d\omega t$$

$$i(\omega t) = \begin{cases} \frac{V_m}{\omega L} (\cos \alpha - \cos \omega t) + \frac{V_{dc}}{\omega L} (\alpha - \omega t) & \text{at } \alpha \leq \omega t \leq \beta \\ 0 & \text{otherwise} \end{cases}$$

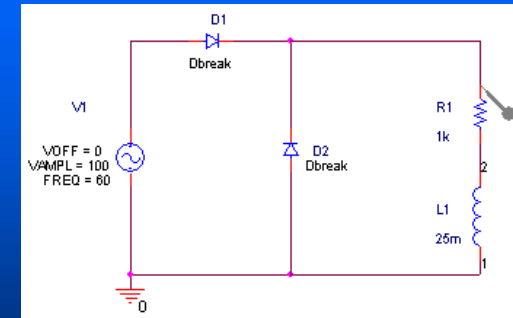


### 3.7 환류 다이오드(Freewheeling Diode)



- ① +인가시 D1를 통하여 L에 전기가 저장
- ② -인가시 L에 저장된 전기가 방전되어 출력을 직류에 가깝도록함

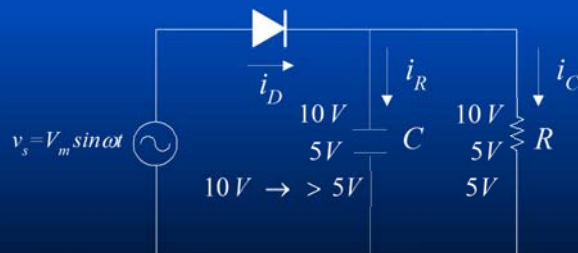
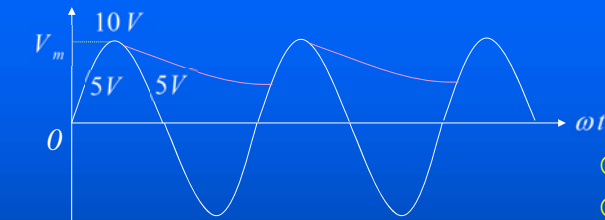
### 예제 3-7 환류다이오드를 가진 반파정류기



#### Parameter

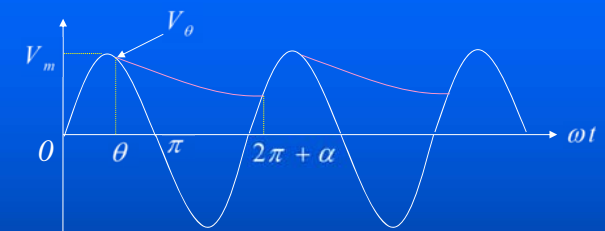
- ✓  $R=2 \Omega$
- ✓  $L=25 \text{ mH}$
- ✓  $f=60 \text{ Hz}$
- ✓  $V_m=100 \text{ V}$
- ✓ Transient:  
0 1m 50m

### 3.8 R-C 반파정류회로



- ① 병렬시 전압이 같음
- ② 10V 인가시 커패시터와 저항이 전압이 10V로 동일
- ③ 전압이 10V에서 5V로 될 때 커패시터의 전압이 10V에서 5V가 되어 방전된 전압이 출력을 직류에 가깝도록함

### 3.8 R-C 반파: 전류(1)



$$V_o(\omega t) = \begin{cases} V_m \sin \omega t & \text{at diode on} \\ V_\theta \cdot e^{-\frac{(\omega t - \theta)}{\omega RC}} & \text{at diode off} \end{cases}$$

\*여기서

$$V_\theta = V_m \sin \theta, \quad \theta = \tan^{-1}(R \cdot \omega C) = -\tan^{-1}(R \cdot \omega C) + \pi$$

### 3.8 R-C 반파: 전류(2)

#### ■ 점화각 (Firing Angle)

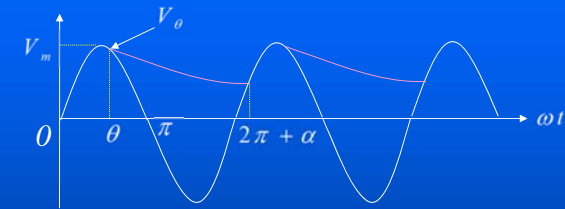
for  $\omega t = 2\pi + \alpha$

$$V_m \sin(2\pi + \alpha) = (V_m \sin \theta) e^{-\frac{(2\pi + \alpha - \theta)}{\omega RC}}$$

#### ■ $\alpha$ 의 계산

- 수치기법(Numerical Analysis) 이용 계산
- OrCAD를 이용하여 계산: 가장 간편함

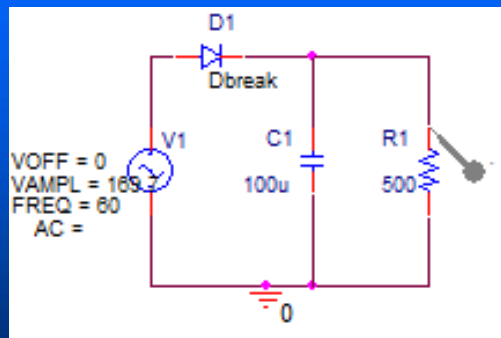
### 3.8 R-C 반파: 전류(3)



■ C의 전류:  $i_C = \omega C \frac{dV_0(\omega t)}{d(\omega t)}$

$$i_C(\omega t) = \begin{cases} \frac{V_m \sin \theta}{R} e^{-\frac{(\omega t - \theta)}{\omega RC}} & \text{at } 0 \leq \omega t \leq 2\pi + \alpha \\ \omega C V_m \cos(\omega t) & \text{at } 2\pi + \alpha \leq \omega t \leq 2\pi + \theta \end{cases}$$

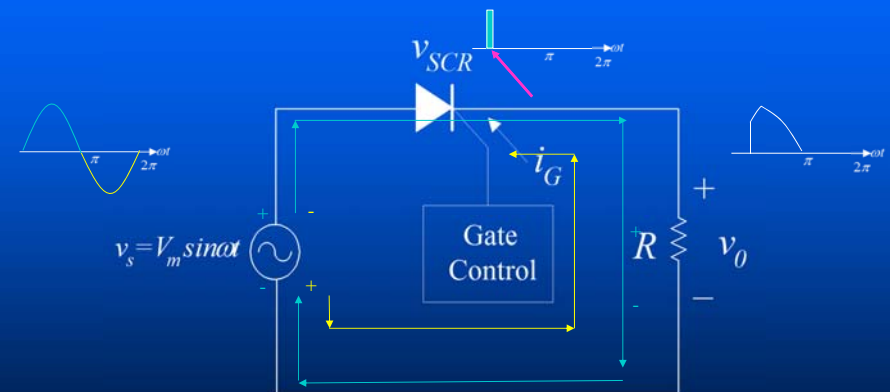
### 예제 3-9 R-C 부하를 갖는 반파정류기



#### ■ Parameter

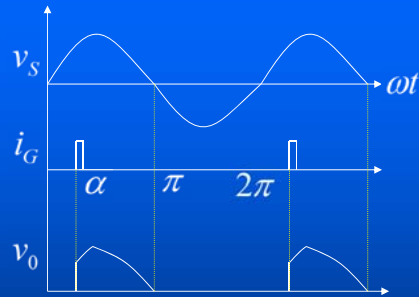
- ✓  $R = 500 \Omega$
- ✓  $C = 100 \mu F$
- ✓  $f = 60 \text{ Hz}$
- ✓  $V_m = 100$
- ✓ Transient: 0 0.01m 50m

### 3.9 반파제어 정류기





### 3.9 반파제어 정류기: 저항부하



■ 전압의 평균값

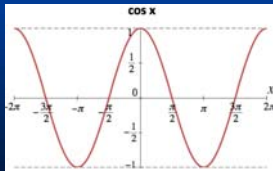
$$V_0 = \frac{1}{2\pi} \int_{\alpha}^{\pi} V_m \sin \omega t d(\omega t)$$

$$\int \sin dt = -\cos t$$

$$= \frac{V_m}{2\pi} [-\cos \omega t]_{\alpha}^{\pi}$$

$$= \frac{V_m}{2\pi} (-\cos \pi + \cos \alpha)$$

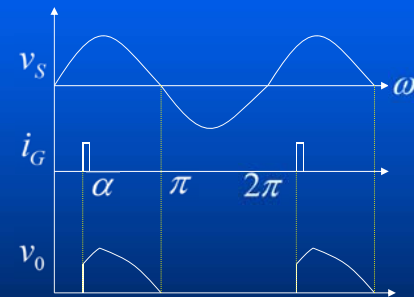
$$= \frac{V_m}{2\pi} (1 + \cos \alpha)$$



### 3.9 반파제어 정류기: 저항부하

■ 전압의 실효값

$$V_{rms} = \sqrt{\frac{1}{2\pi} \int_{\alpha}^{\pi} (V_m \sin \omega t)^2 d(\omega t)}$$



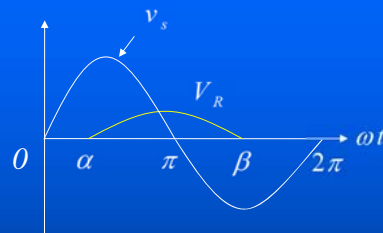
$$\left( \sin^2 \omega t = \frac{1 - \cos 2\omega t}{2} \right)$$

$$= \sqrt{\frac{V_m^2}{2\pi} \left[ \frac{\omega t}{2} - \frac{\sin 2\omega t}{4} \right]_{\alpha}^{\pi}}$$

$$= \sqrt{\frac{V_m^2}{2\pi} \left( \frac{\pi}{2} - \frac{\sin 2\pi}{4} - \frac{\alpha}{2} + \frac{\sin 2\alpha}{4} \right)}$$

$$= \frac{V_m}{2} \sqrt{\left( 1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi} \right)}$$

### 3.9 반파제어 정류기: R-L부하

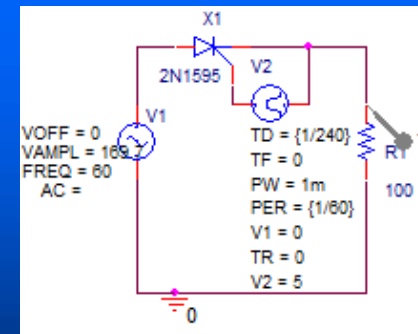


■ 전류

$$i(\omega t) = i_f(\omega t) + i_n(\omega t)$$

$$i(\omega t) = \begin{cases} \frac{V_m}{Z} \left( \sin(\omega t - \theta) - \sin(\alpha - \theta) e^{-\frac{(\alpha - \omega t)}{\omega \tau}} \right) & \text{at } \alpha \leq \omega t \leq \beta \\ 0 & \text{otherwise} \end{cases}$$

### 3.10 OrCAD: 반파제어정류기 설계



■ Parameter

- ✓  $R=1 \text{ k}\Omega$
- ✓  $f=\omega/2\pi=60 \text{ Hz}$
- ✓  $V_m=50 \text{ V}$
- ✓  $V_{pulse}$ 
  - $V1=0 \text{ V}$
  - $V2=5 \text{ V}$
  - $TD=0$
  - $PER=1/60=\{1/60\}$
  - $PW=1 \text{ m}$
- ✓ Transient
  - 0 0.01m 50m

